On The Portfolio Selection Problem

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Doctorado en Ingeniería Matemática

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Outline

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- **The investor has to allocate his budget C to the different risks. Without loss of generality $C = 1$**
- **The investor has many alternatives to invest given by**

$$\mathbf{w} = (\omega_1, \omega_2, \ldots, \omega_n), \quad \sum_{i=1}^{n} \omega_i = 1,\ \omega_i \geq 0,\ i = 1, \ldots, n,$$

where ω_i is the weight (budget proportion) assigned to the risk X_i.

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- **Some answers will be given in this talk**

What is the Problem...?

- **Assume that an investor cares only about the mean and variance of portfolio.**
- **A simple case of two risks**

$$\mathbf{X} = (X_1, X_2) \text{ such that } E(\mathbf{X}) = (\mu_1, \mu_2) \text{ and } \Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}$$

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$$VAR(\mathcal{P}_{\mathbf{w}}) = \omega^2\sigma_1^2 + (1-\omega)^2\sigma_2^2 + 2\omega(1-\omega)\sigma_{12}$$

What is the Problem...?

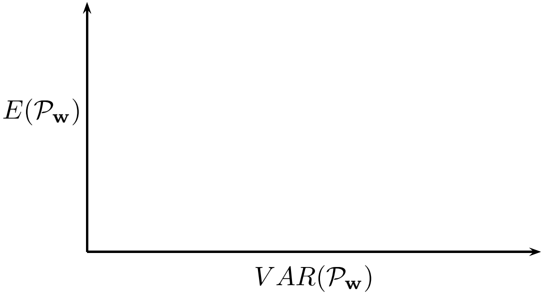
Let $\mathbf{X} = (X_1, X_2)$ **such that**

$\mu_1 = 0.5, \;\; \mu_2 = 0.3, \;\; \sigma_1^2 = 4, \;\; \sigma_2^2 = 1, \;\; \sigma_{12} = 1$. **If $\omega = 1$, then**

$$\mathcal{P}_\mathbf{w} = \omega X_1 + (1-\omega)X_2 = X_1$$

$$E(\mathcal{P}_\mathbf{w}) = \omega\mu_1 + (1-\omega)\mu_2 = 0.5$$

$$VAR(\mathcal{P}_\mathbf{w}) = \omega^2\sigma_1^2 + (1-\omega)^2\sigma_2^2 + 2\omega(1-\omega)\sigma_{12} = 4$$



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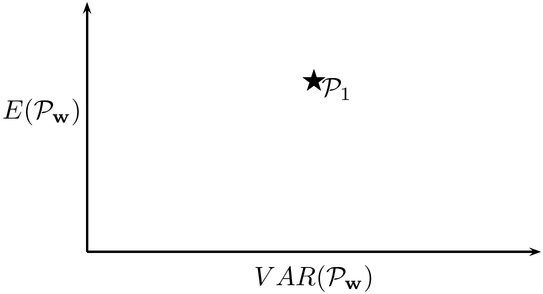

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$\mu_1 = 0.5,\ \ \mu_2 = 0.3,\ \ \sigma_1^2 = 4,\ \ \sigma_2^2 = 1,\ \ \sigma_{12} = 1$. **If $\omega = 0$, then**

$$\mathcal{P}_{\mathbf{w}} = \omega X_1 + (1-\omega)X_2 = X_2$$
$$E(\mathcal{P}_{\mathbf{w}}) = \omega\mu_1 + (1-\omega)\mu_2 = 0.3$$
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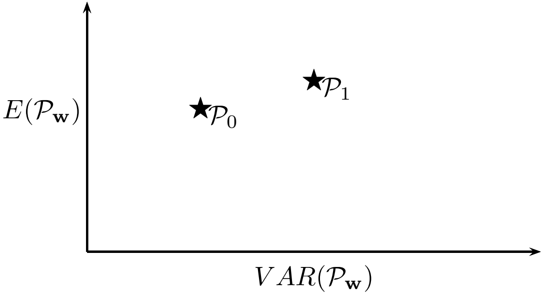

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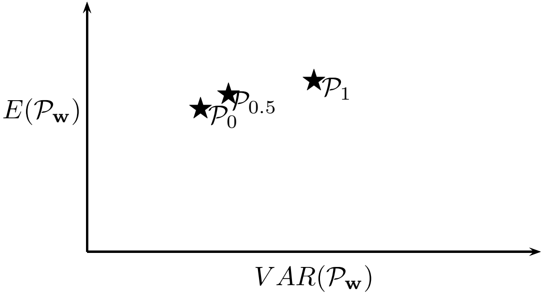
$\mu_1 = 0.5, \ \ \mu_2 = 0.3, \ \ \sigma_1^2 = 4, \ \ \sigma_2^2 = 1, \ \ \sigma_{12} = 1$. **If** $\omega = 0.5$, **then**

$$\mathcal{P}_{\mathbf{w}} = \omega X_1 + (1-\omega) X_2 = 0.5 X_1 + 0.5 X_2$$
$$E(\mathcal{P}_{\mathbf{w}}) = \omega \mu_1 + (1-\omega)\mu_2 = 0.75$$
$$VAR(\mathcal{P}_{\mathbf{w}}) = \omega^2 \sigma_1^2 + (1-\omega)^2 \sigma_2^2 + 2\omega(1-\omega)\sigma_{12} = 0.87$$



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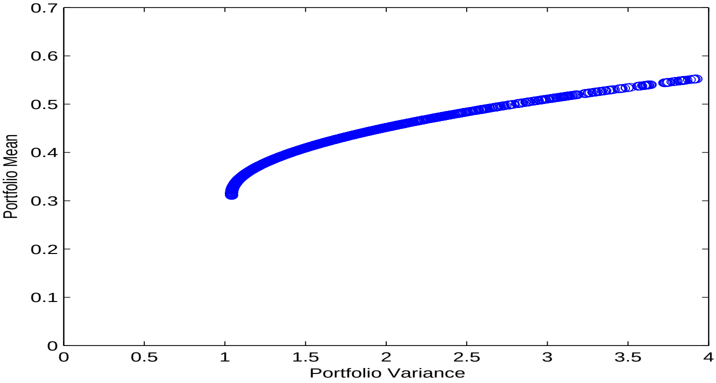

Figure: Mean-Variance for Different ω Values.

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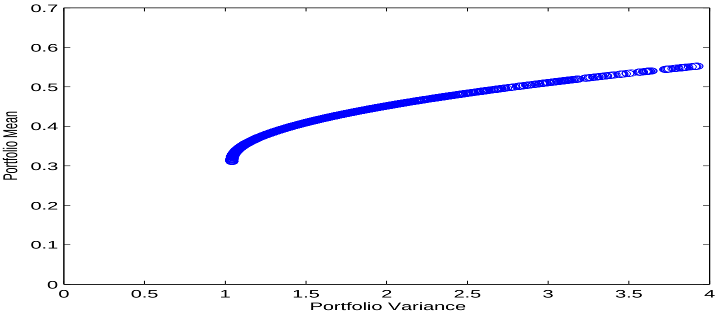

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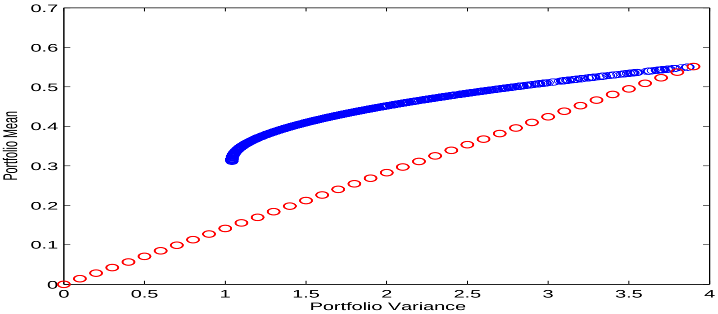

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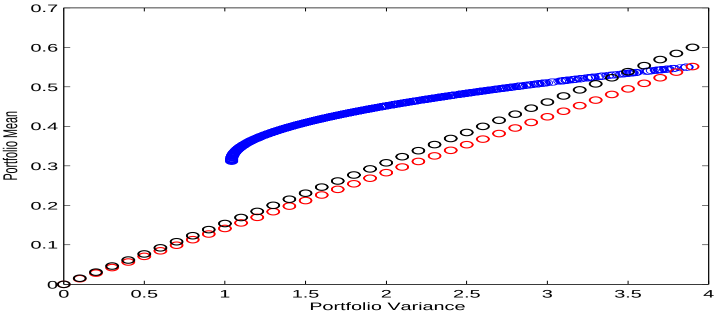

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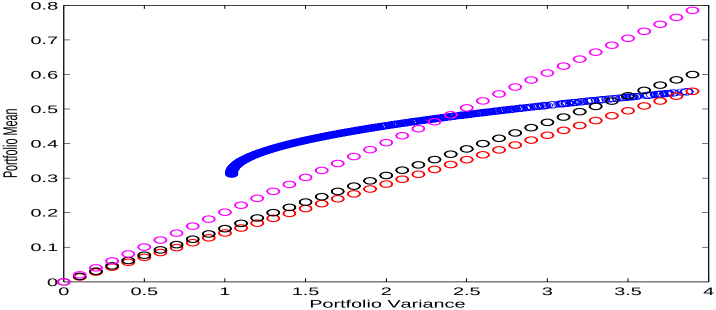

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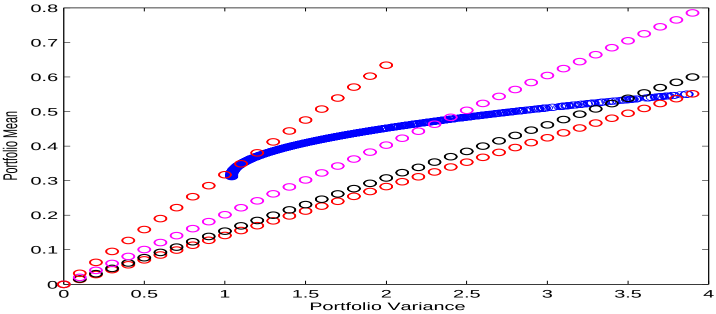

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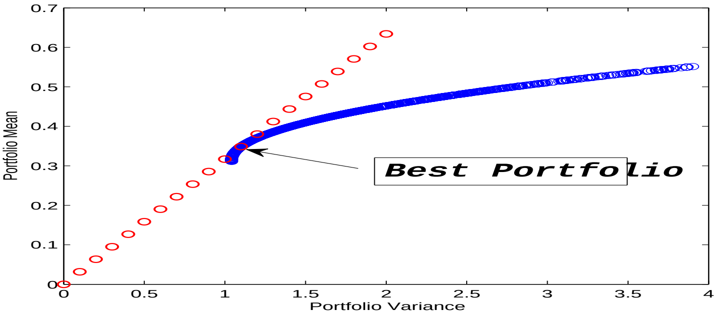

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Efficient Frontier

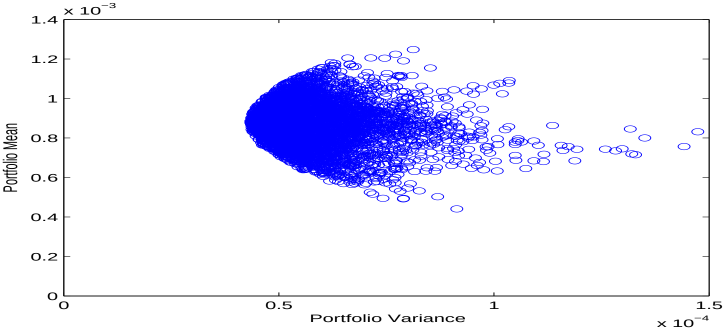
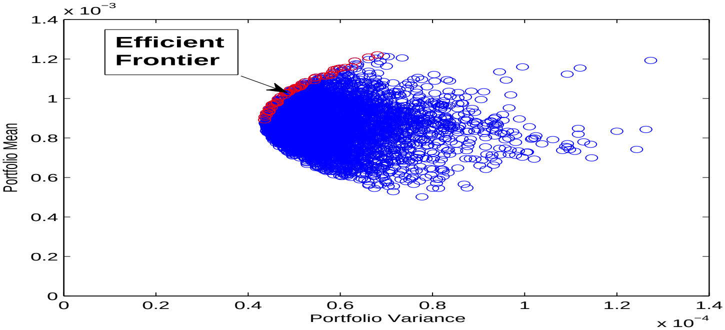


Figure: Feasible Portfolios

The set of couples risk-return that cannot be improved at the same time is called **Efficient Frontier.** Markowitz (1952)

Efficient Frontier



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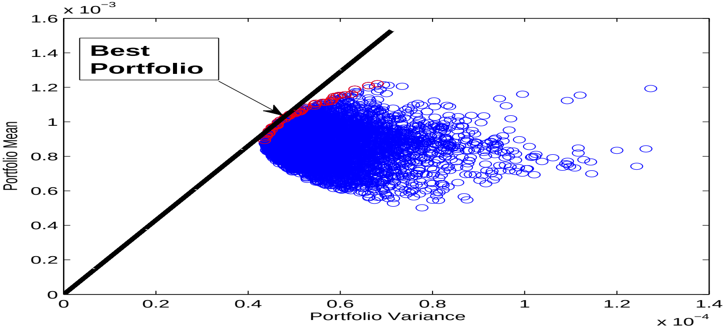

Figure: Best Portfolio

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Portfolio Problem

- **Consider the random vector $\mathbf{X} = (X_1, X_2, \ldots, X_n)$ and the Portfolio Random Variable**

$$\mathcal{P}_{\mathbf{w}} = \sum_{i=1}^{n} \omega_i X_i$$

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- **Let $\mathfrak{U}$ be his/her subjective utility function. Assume that $\mathfrak{U}' \geq 0$ and $\mathfrak{U}'' \leq 0$.** **Increasing and Concave**
- **The portfolio problem in this case is given by**

$$\max_{\mathbf{w}} \; E\mathfrak{U}\left(\mathcal{P}_{\mathbf{w}}\right) \quad \textbf{s.t.} \quad \sum_{i=1}^{n} \omega_i = 1.$$

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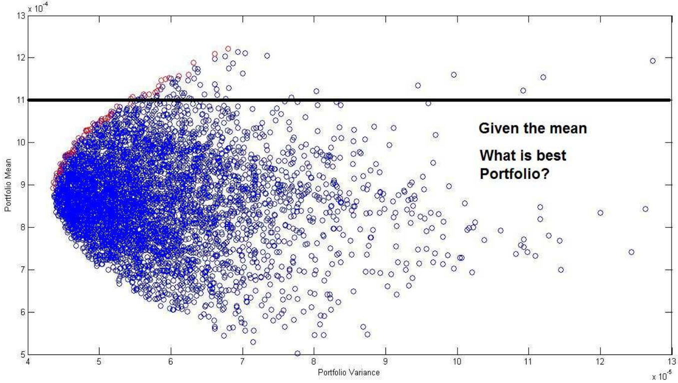
Efficient Frontier



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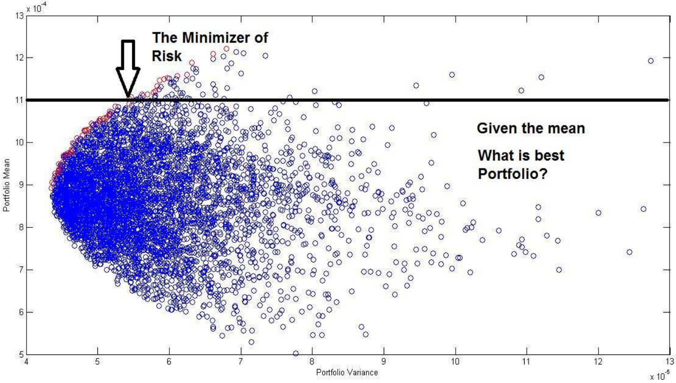

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Minimum- Variance Portfolio

Markowitz (1952)

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Given the mean-value the best portfolio is the solution to the optimization problem.

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}'\Sigma\mathbf{w} \\ \text{s.t.} \quad & E(\mathcal{P}_{\mathbf{w}}) = \mu \\ & \sum_{i=1}^{n} \omega_i = 1 \end{aligned}$$

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If you have data you can use estimators for Σ and $E(X_i)$.

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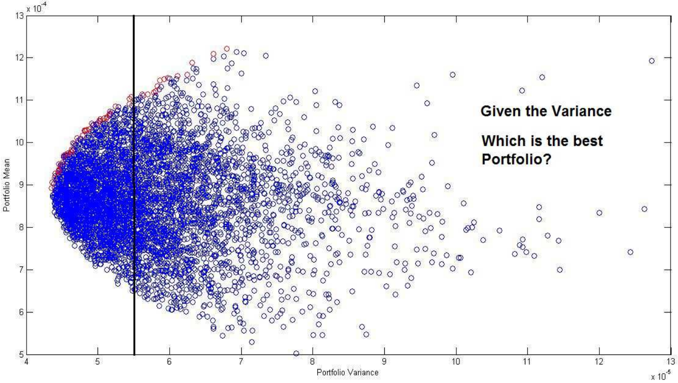

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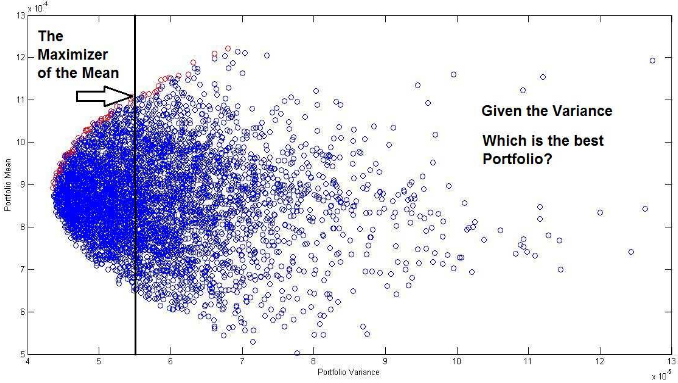

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Maximum-Mean Portfolio

Following Markowitz Model (1952) this portfolio also will be on the efficient frontier. Therefore, given the variance, the best portfolio is the solution to the optimization problem

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$$\max_{\mathbf{w}} \; E(\mathcal{P}_{\mathbf{w}})$$
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Mean-Variance Portfolio

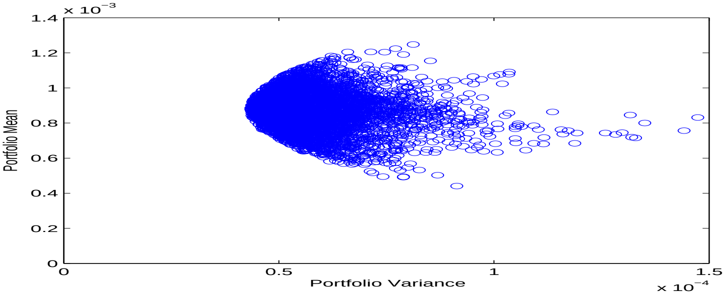
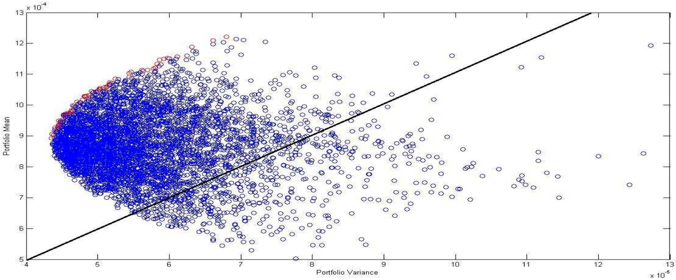


Figure: Best Portfolio

$$\max_{\mathbf{w}} \ \mathbf{w}'\Sigma\mathbf{w} - \frac{1}{\alpha}E(\mathcal{P}_{\mathbf{w}})$$

s.t. $\sum^{n} \omega_i = 1.$ α is the risk-aversion parameter

Mean-Variance Portfolio



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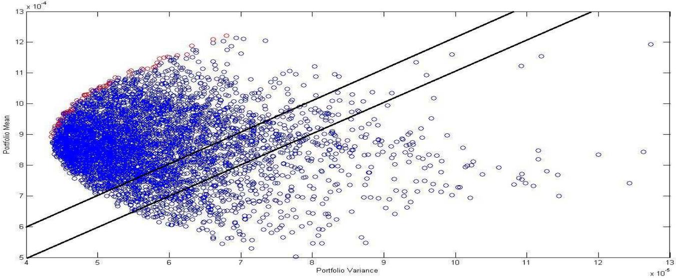

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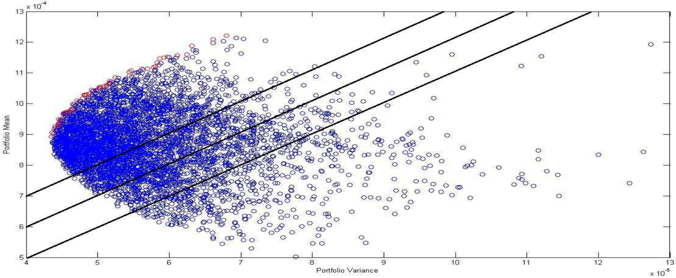

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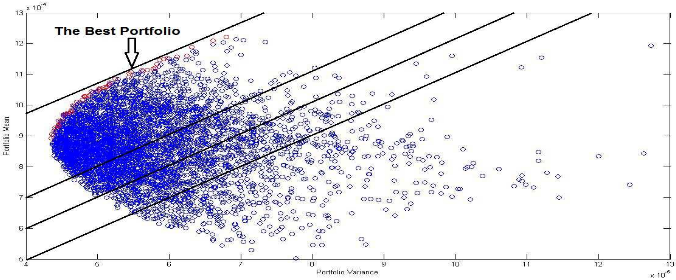

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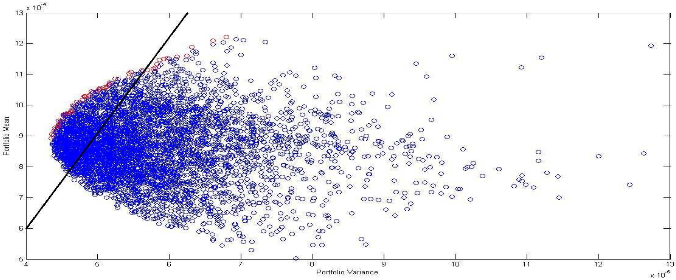

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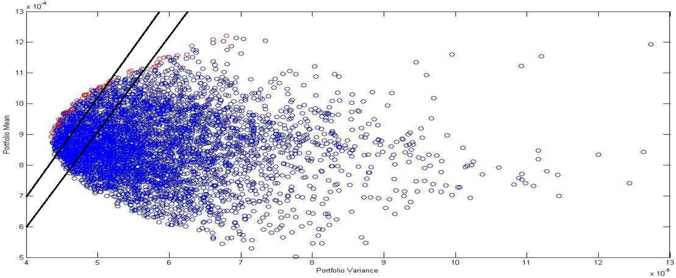

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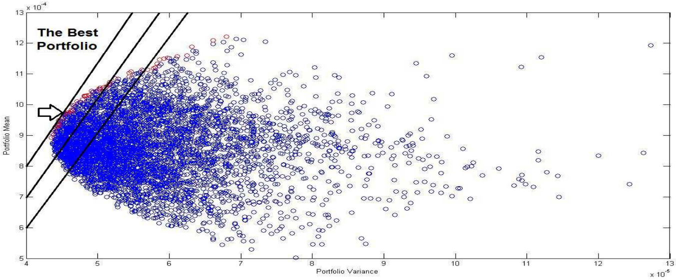

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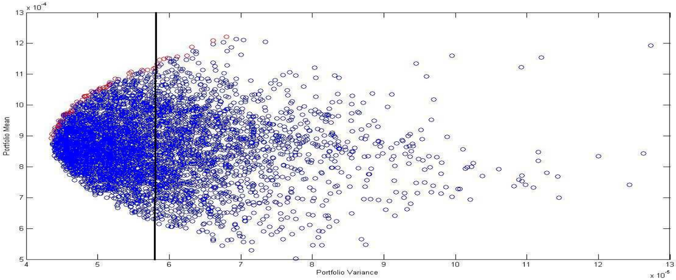

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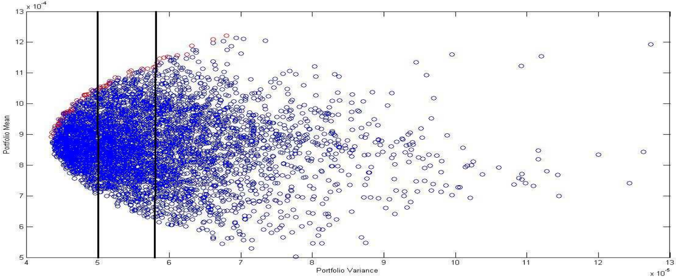

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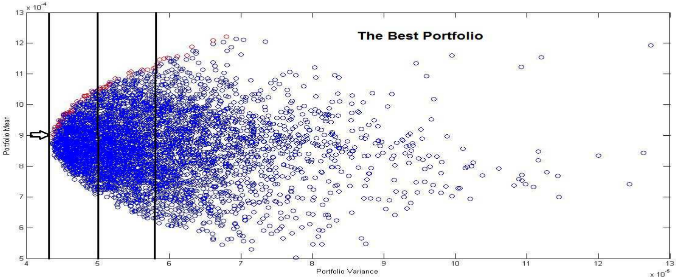

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Risk Inverse Weighting Analysis PIR

Puerta and Laniado (2010)

Let $\mathbf{X} = (X_1, \ldots, X_n)$ be a risky assets vector.

$$\mathcal{P}_{\mathbf{w}} = \sum_{i=1}^{n} \omega_i X_i, \quad \omega_i = \frac{\frac{1}{\rho(X_i)}}{\sum_{i=1}^{n} \frac{1}{\rho(X_i)}}$$

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Less Weight to Higher Risk

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- **if $\mathbf{X} = (X_1, \ldots, X_n)$ is exhangeable, then PIR$\equiv \frac{1}{n}$-rule**
- **if $\mathbf{X} = (X_1, \ldots, X_n)$ is comonotonic and ρ is comonotonic risk measure, then the risk of PIR is smaller than the risk of $\frac{1}{n}$-rule.**

Risk Inverse Weighting Analysis

DeMiguel et al. (2009) and Muller and Stoyan (2002) showed the advantages of using $\frac{1}{n}$-rule (Naive Portfolio).

$$\mathcal{P}_{\mathbf{w}} = \sum_{i=1}^{n} \omega_i X_i = \sum_{i=1}^{n} \frac{1}{n} X_i$$

- **if $\mathbf{X} = (X_1, \ldots, X_n)$ is exhangeable, then PIR$\equiv \frac{1}{n}$-rule**
- **if $\mathbf{X} = (X_1, \ldots, X_n)$ is comonotonic and ρ is comonotonic risk measure, then the risk of PIR is smaller than the risk of $\frac{1}{n}$-rule.**
- **if $\mathbf{X} = (X_1, X_2)$, the variance of PIR is smaller than the variance of $\frac{1}{n}$-rule.**

Indice

Portfolio Problem

- **Consider the random vector $\mathbf{X} = (X_1, X_2, \ldots, X_n)$ and the Portfolio Random Variable**

$$\mathcal{P}_{\mathbf{w}} = \sum_{i=1}^{n} \omega_i X_i$$

- **Let $\mathfrak{U}$ be his/her subjective utility function. Assume that $\mathfrak{U}' \geq 0$ and $\mathfrak{U}'' \leq 0$. Increasing and Concave**
- **The portfolio problem in this case is given by**

$$\max_{\mathbf{w}} \; E\mathfrak{U}\left(\mathcal{P}_{\mathbf{w}}\right) \quad \textbf{s.t.} \qquad \sum_{i=1}^{n} \omega_i = 1.$$

Portfolio Problem

$$\max_{\mathbf{w}} \; EU\left(\mathcal{P}_{\mathbf{w}}\right) \quad \text{s.t.} \quad \sum_{i=1}^{n} \omega_i = 1. \tag{1}$$

Hadar and Russel (1971)

Investigated the problem (1) for iid random variables in the bivariate case. They showed that the solution to the problem (1) is the $\frac{1}{n}$-rule

$$\mathcal{P}_{\mathbf{w}}^{*} = \mathcal{P}_{\frac{1}{2}}$$

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$$\mathcal{P}^*_{\mathbf{w}} = \mathcal{P}_{\frac{1}{2}}$$

Ma (2000)

Showed that if $(X_1, X_2, \ldots, X_n)$ are exchangeable . Then the solution of (1) is the $\frac{1}{n}$-rule.

$$\mathcal{P}^*_{\mathbf{w}} = \mathcal{P}_{\frac{1}{n}}$$

Portfolio Problem

$$\max_{\mathbf{w}} \; EU\left(\mathcal{P}_{\mathbf{w}}\right) \quad \text{s.t.} \qquad \sum_{i=1}^{n} \omega_i = 1. \qquad (2)$$

Pellerey and Semeraro (2005)

They considered $\mathbf{X} = (X_1, X_2)$**,** $S = X_1 + X_2$ **and** $D = X_2 - X_1$**.**
They showed that if (S, D) **is** PQD **and** $E(X_2) \leq E(X_1)$**, then**

$$EU\left[(1-\alpha)X_1 + \alpha X_2\right]$$

is decreasing in $\alpha \in [\frac{1}{2}, 1]$.
The solution to the problem (2) is the $\frac{1}{n}$**-rule**

$$\mathcal{P}^*_{\mathbf{w}} = \mathcal{P}_{\frac{1}{2}}$$

Portfolio Problem

Laniado et al. (2012)

Consider $\mathbf{X} = (X_1, X_2)$ and assume that there is a vector $\mathbf{u} = (u_1, u_2)$ with $\| \mathbf{u} \| = 1$. If

$$\begin{pmatrix} u_1 & u_2 \\ -u_2 & u_1 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \text{ is } \quad PQD \quad \text{and} \quad u_1 E(X_2) - u_2 E(X_1) \leq 0.$$

$$E\left[U\left(\frac{\sqrt{2}}{2}\left(u_1 + u_2 - 2u_2\alpha\right) X_1 + \frac{\sqrt{2}}{2}\left(2u_1\alpha - u_1 + u_2\right) X_2 \right) \right]$$

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is decreasing in $\alpha \in [\frac{1}{2}, 1]$.

$$\mathcal{P}^*_{\mathbf{w}} = \frac{\sqrt{2}}{2} u_1 X1 + \frac{\sqrt{2}}{2} u_2 X2$$

Elliptical Distributions

Definition

The random vector $\mathbf{X} = (X_1, \ldots, X_n)'$ is said to have an *elliptical distribution* with parameters μ and $\mathbf{\Sigma}$ if its characteristic function can be expressed as

$$E[\exp(i\mathbf{t}'X)] = \exp(i\mathbf{t}'\mu)\phi\left(\mathbf{t}'\mathbf{\Sigma}\mathbf{t}\right), \quad \mathbf{t} = (t_1, \ldots, t_n)', \tag{3}$$

for some function ϕ, and if $\mathbf{\Sigma}$ is such that $\mathbf{\Sigma} = \mathbf{A}\mathbf{A}'$ for some matrix $\mathbf{A}(n \times m)$.

Property

Laniado et al. (2012)

Let $\mathbf{X} = (X_1, X_2)$ be a random vector elliptically distributed with parameters $\mu = 0$ and $\Sigma_{\mathbf{X}}$. Then there exists a rotation matrix such that $\mathcal{R}\mathbf{X}$ is exchangeable.

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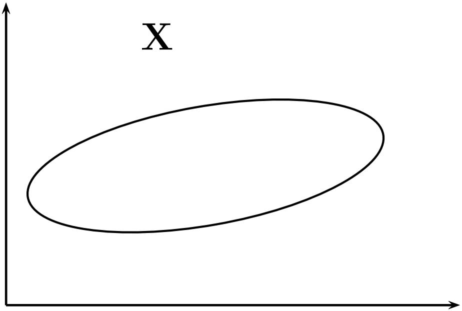
Let $\mathbf{X} = (X_1, X_2)$ be a random vector elliptically distributed with parameters $\mu = 0$ and $\Sigma_{\mathbf{X}}$. Then there exists a rotation matrix such that $\mathcal{R}\mathbf{X}$ is exchangeable.

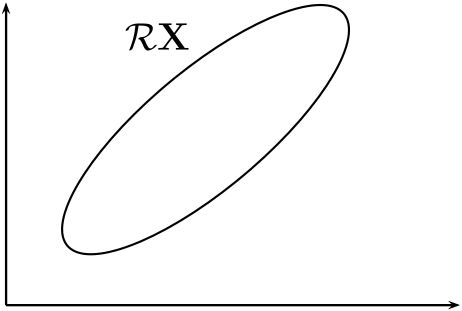
$$\mathcal{R} = \frac{\sqrt{2}}{2} \begin{pmatrix} q_{11} + q_{21} & q_{21} - q_{11} \\ q_{11} - q_{21} & q_{11} + q_{21} \end{pmatrix}.$$

$\Sigma_{\mathbf{X}} = QDQ'$ **and** $Q = (q_{ij})$

Elliptical Distribution



Rotated Elliptical Distribution



Theorem 3.A.35. Shaked and Shanthikumar (2007)

Let $X_1, \ldots, X_n$ be exchangeable random variables. Let $\mathbf{a} = (a_1, \ldots, a_n)'$ and $\mathbf{b} = (b_1, \ldots, b_n)'$ such that $\mathbf{a} \prec \mathbf{b}$, then

$$\sum_{i=1}^{n} a_i X_i \geq_{cv} \sum_{i=1}^{n} b_i X_i$$

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Elliptical Distribution, $n > 2$

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Let $\mathbf{X} = (X_1, \ldots, X_n)'$ be a random vector elliptically distributed with parameters $\mu_{\mathbf{X}} = 0$ and $\mathbf{\Sigma}_{\mathbf{X}}$ is such that it has at least $n-1$ equal eigenvalues given by $\lambda_1 \geq \lambda_2 = \cdots = \lambda_n = \lambda > 0$. Then there exists a rotation matrix $\mathcal{R}$ such that $\mathcal{R}\mathbf{X}$ has exchangeable components.

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Portafolio Comparison

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$$X \leq_{st} Y \Longleftrightarrow E[\phi(X)] \leq E[\phi(Y)],$$

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Therefore, given the portfolios $\mathcal{P}_{\omega_1}$ and $\mathcal{P}_{\omega_2}$ such that

$$\mathcal{P}_{\omega_1} \leq_{icv} \mathcal{P}_{\omega_2},$$

then an investor with increasing and concave utility function prefers $\mathcal{P}_{\omega_2}$.

Portfolio Problem

$$\max_{\vec{\omega}} \; E\mathfrak{U}\left(\mathcal{P}_{\omega}\right) \quad \text{s.t.} \quad \sum_{i=1}^{n} \omega_i = 1. \tag{4}$$

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If $X_1, \ldots, X_n$ are independent with

$$X_1 \geq_{lr} X_2 \geq_{lr} \cdots \geq_{lr} X_n,$$

and $\mathfrak{U}$ is increasing. Then the optimization problem (4) has an optimal solution with $\omega_1 \geq \omega_2 \geq \cdots \geq \omega_n$.

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$$X \leq_{lr} Y \iff \frac{f_Y(t)}{f_X(t)} \uparrow_t$$

Portfolio Problem

$$\max_{\vec{\omega}} \; E\mathfrak{U}\left(\mathcal{P}_\omega\right) \quad \text{s.t.} \qquad \sum_{i=1}^{n} \omega_i = 1. \tag{5}$$

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If $X_1, \ldots, X_n$ are independent with

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and $\mathfrak{U}$ is increasing and concave. Then the optimization problem (5) has an optimal solution with. $\omega_1 \geq \omega_2 \geq \cdots \geq \omega_n.$

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Indice

Alternative efficient frontiers

Let Θ be a set of k criteria for evaluating the performance of the portfolio.
In the classical Markowitz model $k = 2$ and corresponds to mean and variance of the portfolio. Consider any criterion $c_i \in \Theta$, $i = 1, \ldots, k$ and denote.

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$$\theta_{c_i} = \begin{cases} 1 & \text{if the investor wants a portfolio with a low value of the criterion } c_i \\ -1 & \text{if the investor wants a portfolio with a high value of the criterion } c_i \end{cases}$$

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For example, if

$$\Theta = \{\textbf{return},\ \textbf{risk},\ \textbf{Sharpe-ratio},\ \textbf{entropy}\} = \{c_1, c_2, c_3, c_4\},$$

then

$$\theta_{\text{return}} = \theta_{c_1} = -1, \quad \theta_{\text{risk}} = \theta_{c_2} = 1, \quad \theta_{\text{Sr}} = \theta_{c_3} = -1, \quad \theta_{\text{entropy}} = \theta_{c_4} = -1.$$

Alternative efficient frontier

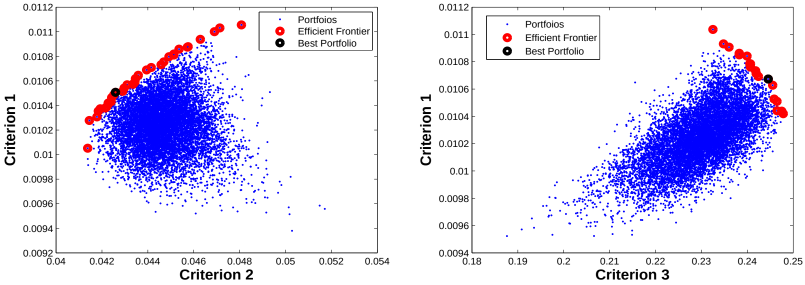
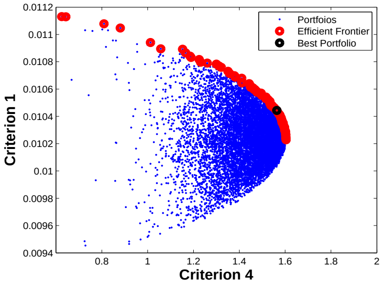
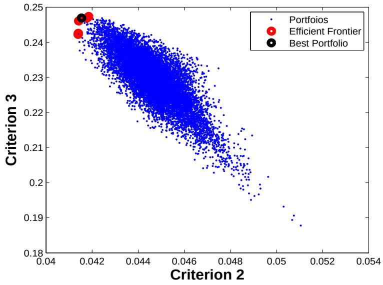


Figure: $\mathbf{u} = \frac{1}{\sqrt{2}}[1, -1]'$ $\quad$ $\mathbf{u} = \frac{1}{\sqrt{2}}[-1, -1]'$

Criterion 1	**Returns**	-1
Criterion 2	**Variance**	1
Criterion 3	**Sharpe ratio**	-1
Criterion 4	**Entropy**	-1

Alternative efficient frontier





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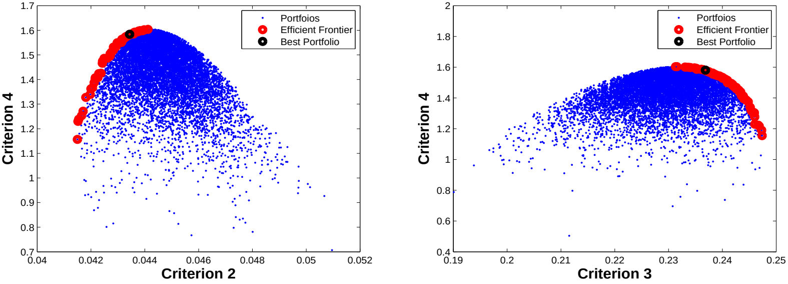

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Alternative efficient frontier

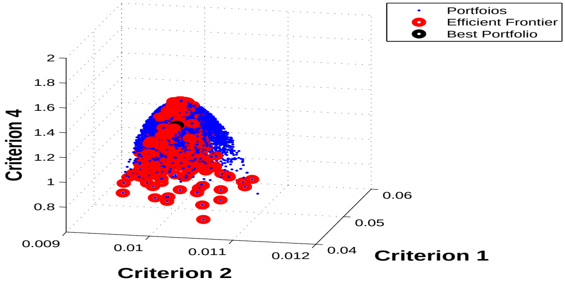

Figure: $\mathbf{u} = \frac{1}{\sqrt{3}}[-1, 1, -1]'$

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Portfolio selection under extremality

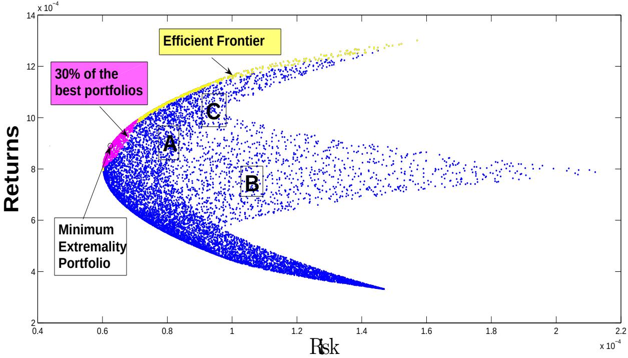

Figure: **Feasible Portfolios**

Application to real data

Table: **Portfolios notation in this work**

Criteria Portfolio notation	returns and variance P_{12}	returns and Sharpe ratio P_{13}
Criteria Portfolio notation	returns and entropy P_{14}	variance and Sharpe ratio P_{23}
Criteria Portfolio notation	variance and entropy P_{24}	Sharpe ratio and entropy P_{34}

Table: **Portfolios notation for comparisons**

$\frac{1}{n}$	Equally-weighted Portfolio
MEAN	Mean-variance portfolio with shortsales constrained
MEANU	Mean-Variance portfolio with shortsales unconstrained
MIN	Minimum-Variance portfolio with shortsales constrained
MINU	Minimum-Variance portfolio with shortsales unconstrained

Results

Test proposed by Memmel (2003). **$\frac{1}{n}$-rule is a good benchmark DeMiguel et al. (2009b)**

Table: **Portfolio Sharpe ratios**

Strategy	5Spain	6Spain	10Spain	25Spain	40Spain	48Ind	8Indexes
in this work							
P_{12}	0.7218 (0.6948)	0.5333 (0.1315)	0.5498 (0.0418)	0.5006 (0.0314)	0.3700 (0.0956)	0.2929 (0.0965)	0.1070 (0.3158)
P_{13}	0.7478 (0.6084)	0.5279 (0.1399)	0.5989 (0.0378)	0.5056 (0.0854)	0.4044 (0.0179)	0.2789 (0.5170)	0.1003 (0.4829)
P_{14}	0.7196 (0.6466)	0.4391 (0.0519)	0.4438 (0.2303)	0.4558 (0.0978)	0.3564 (0.0819)	0.2793 (0.3309)	0.0896 (0.8759)
P_{23}	0.7080 (0.9093)	0.4962 (0.2988)	0.5375 (0.1723)	0.5406 (0.0178)	0.3166 (0.5215)	0.2801 (0.4466)	0.0985 (0.5582)
P_{24}	0.6941 (0.8454)	0.3446 (0.3012)	0.3656 (0.7308)	0.4735 (0.0610)	0.3182 (0.5137)	0.2836 (0.1533)	0.0848 (0.6856)
P_{34}	0.7114 (0.6893)	0.4308 (0.1397)	0.4881 (0.0025)	0.4514 (0.0198)	0.3766 (0.0204)	0.2731 (0.8809)	0.0910 (0.7383)
for comparison							
1/n	0.6997	0.3753	0.3815	0.3791	0.2955	0.2719	0.0883
MEAN	0.4132 (0.0750)	0.0804 (0.1902)	0.1075 (0.1999)	0.2213 (0.4145)	−0.1400 (0.0024)	0.2296 (0.4806)	0.0555 (0.7131)
MEANU	0.6632 (0.7598)	0.4750 (0.3314)	0.5354 (0.1060)	0.4201 (0.8452)	0.1960 (0.6209)	0.0921 (0.0519)	−0.0267 (0.4246)
MIN	0.6502 (0.5314)	0.1373 (0.2605)	0.2745 (0.5303)	0.2881 (0.5073)	0.3500 (0.5276)	0.2293 (0.4326)	0.0961 (0.8968)
MINU	0.6199 (0.4932)	0.0871 (0.1989)	0.2577 (0.4981)	−0.1271 (0.0276)	0.0012 (0.0948)	0.1123 (0.0393)	−0.0426 (0.0640)

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MEANU	0.6632 (0.7598)	0.4750 (0.3314)	0.5354 (0.1060)	0.4201 (0.8452)	0.1960 (0.6209)	0.0921 (0.0519)	−0.0267 (0.4246)
MIN	0.6502 (0.5314)	0.1373 (0.2605)	0.2745 (0.5303)	0.2881 (0.5073)	0.3500 (0.5276)	0.2293 (0.4326)	0.0961 (0.8968)
MINU	0.6199 (0.4932)	0.0871 (0.1989)	0.2577 (0.4981)	−0.1271 (0.0276)	0.0012 (0.0948)	0.1123 (0.0393)	−0.0426 (0.0640)

Results

Test proposed by Memmel (2003). $\frac{1}{n}$-rule is a good benchmark DeMiguel et al. (2009b)

Table: **Portfolio Sharpe ratios**

Strategy	5Spain	6Spain	10Spain	25Spain	40Spain	48Ind	8Indexes
in this work							
P_{12}	0.7218 (0.6948)	0.5333 (0.1315)	0.5498 (0.0418)	0.5006 (0.0314)	0.3700 (0.0956)	0.2929 (0.0965)	0.1070 (0.3158)
P_{13}	0.7478 (0.6084)	0.5279 (0.1399)	0.5989 (0.0378)	0.5056 (0.0854)	0.4044 (0.0179)	0.2789 (0.5170)	0.1003 (0.4829)
P_{14}	0.7196 (0.6466)	0.4391 (0.0519)	0.4438 (0.2303)	0.4558 (0.0978)	0.3564 (0.0819)	0.2793 (0.3309)	0.0896 (0.8759)
P_{23}	0.7080 (0.9093)	0.4962 (0.2988)	0.5375 (0.1723)	0.5406 (0.0178)	0.3166 (0.5215)	0.2801 (0.4466)	0.0985 (0.5582)
P_{24}	0.6941 (0.8454)	0.3446 (0.3012)	0.3656 (0.7308)	0.4735 (0.0610)	0.3182 (0.5137)	0.2836 (0.1533)	0.0848 (0.6856)
P_{34}	0.7114 (0.6893)	0.4308 (0.1397)	0.4881 (0.0025)	0.4514 (0.0198)	0.3766 (0.0204)	0.2731 (0.8809)	0.0910 (0.7383)
for comparison							
1/n	0.6997	0.3753	0.3815	0.3791	0.2955	0.2719	0.0883
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thanks for your attention