An approach to the study of time, time-frequency and time-scale transformations for seismic migration problems

Juan Guillermo Paniagua

Advisor: Olga Lucía Quintero M.

EAFIT University
Inspire, Create, Transform

Content

Antecedents

Research project ECOPETROL-COLCIENCIAS

Seismic pre-stack migration in depth by extrapolating wave fields using high performance computing for massive data in complex areas.

Cooperative research project: Universidad de Antioquia, Instituto Tecnológico Metropolitano -ITM, Universidad Industrial de Santander, Universidad de Pamplona.

Antecedents

Challenges in the oil industry

- Minimizing exploration costs.
- Minimize the degree of uncertainty in exploration.
- Improve subsurface characterization.
- Deepwater oil reservoirs.
- Deep reservoirs and complex areas.
- Small reservoirs in known areas.

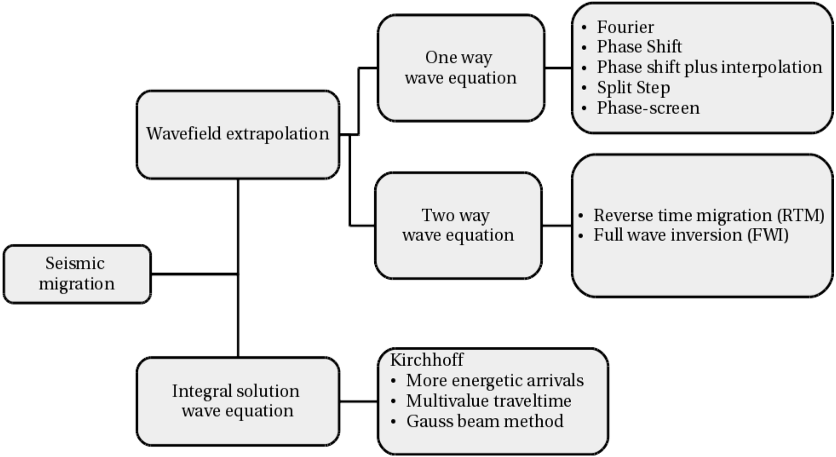
Antecedents

Marine seismic acquisition

https://youtu.be/ZesI8PevfAQ

Antecedents

Seismic migration



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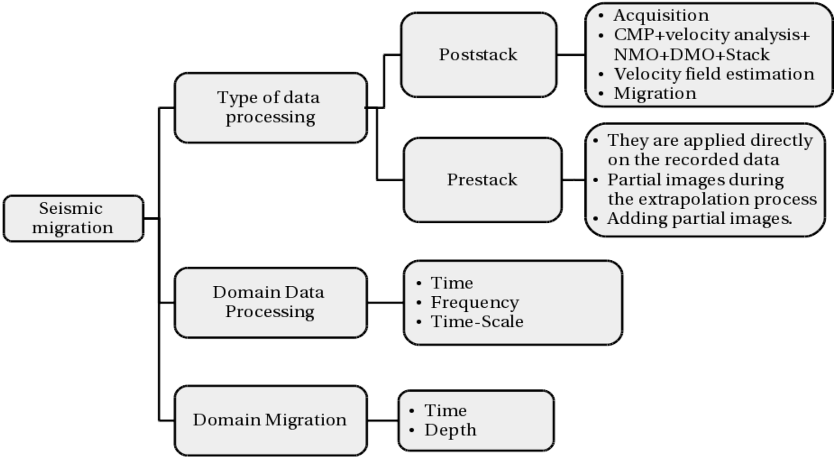

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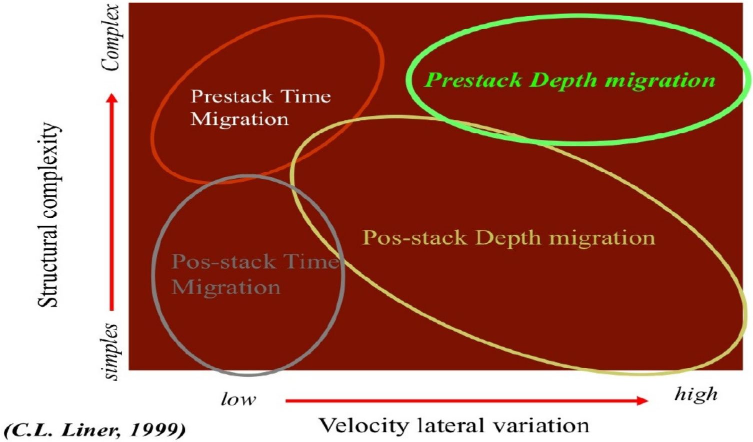

Background

- Geometric migration(until 1960)[1].
- Diffraction summation migration (or diffraction stack)[2].
- Finite difference schemes for hyperbolic equations[3].

Hagedoorn, 1954, [40]
Schneider, 1971, [52]
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- Kirchhoff migration[4].
- Reverse time migration[5].
- Kirchhoff migration enhanced the amplitudes and phases[6].

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- Migration by Fourier transform. (Migration in f-k domain)[7].
- Phase shift migration[8].
- Phase shift plus interpolation (PSPI migration)[9].
- Split step migration[10].

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8 Gazdag, 1978, [34]
9 Gazdag and Sguazzero, 1984, [35]
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Background

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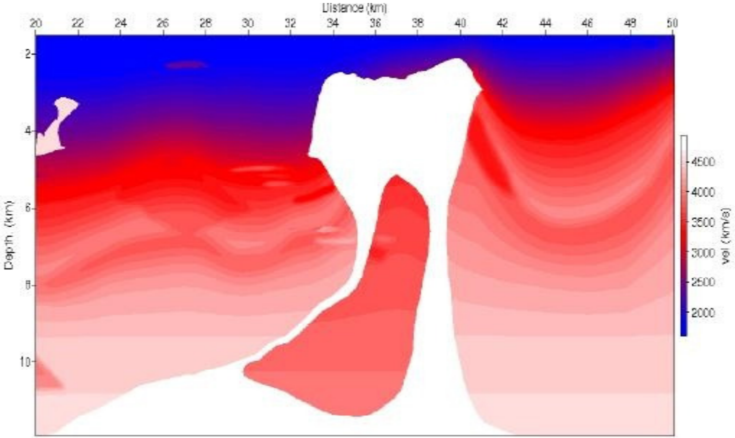
In recent years there have been extensions of these methods to three dimensions and pre-stack migration, with further refinements in terms of accuracy and efficiency.

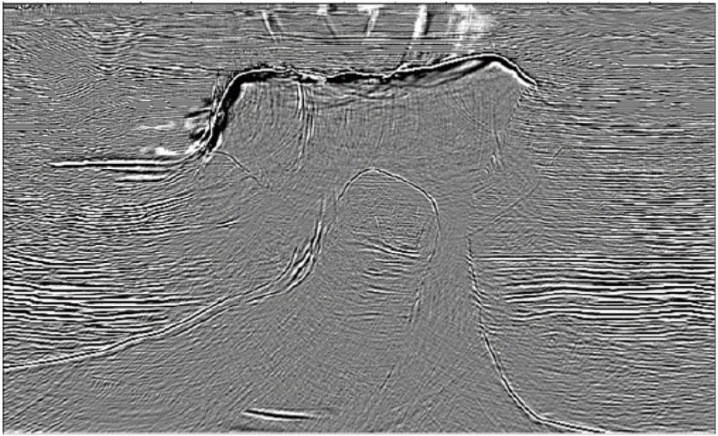
Example in migration

BP model 2007



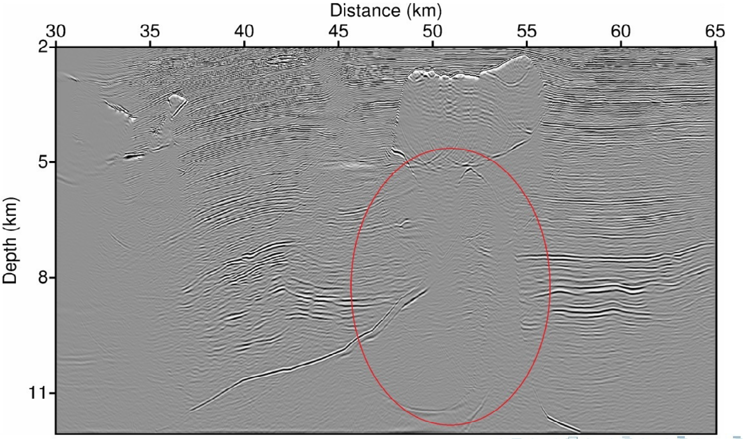
Examples in migration

Kirchhoff migration



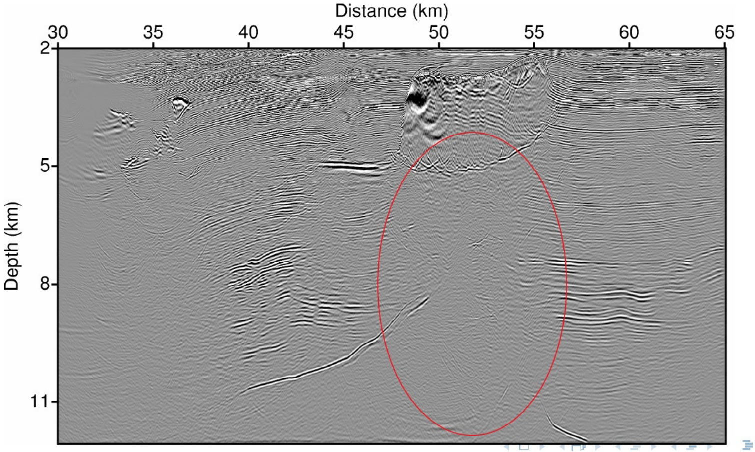
Examples in migration

PSPI migration



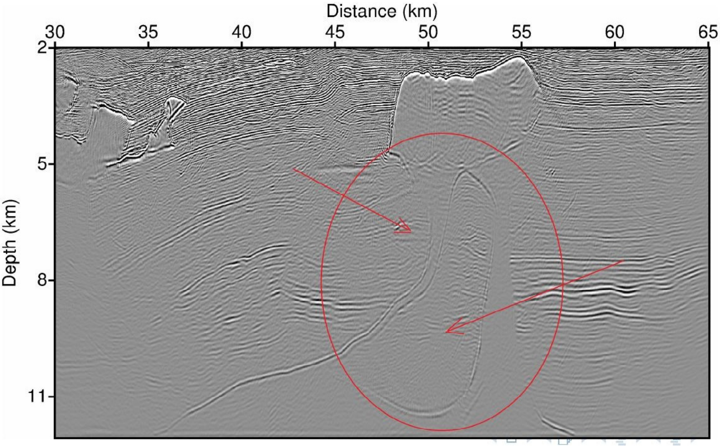
Examples in migration

Split-Step migration



Examples in migration

Reverse time migration



Background

Wavelet analysis

Methods for the analysis in the frequency-time domain (scale). Analysis of non-stationary signals or strong changes in small intervals. They are used in:

- Geophysics (Chakraborty, 1995, [9], Foufoula-Georgiou y Kumar, 1995, [28])
- Astrophysics (Starck, 2010, [56])
- Biology (Meyers,1993, [48])
- Signal and image in Medicine (Burt, 1989, [7], Polikar, 1997, [51])

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- Satellite images (Demirel, 2010, [16])
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A brief history

- Gabor transform,1946[11].
- Gabor transform modified with dilated Windows[12].
- Morlet, (1982).Morlet wavelet basis[13].
- Goupillaud (1984) [14].

Gabor, 1946, [30]
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- Seismic data compression[17]and satellite transmission [18].
- Emergence of new orthogonal wavelet transforms[19].

Daubechies, 1988,[15]
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- Complex seismic trace analysis[23].
- STFT, CWT, MPD [24].
- Acoustic wavelet transform[25].
- Fast Kirchhoff migration[26].

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- Study of seismic migration methods.
- Generation of 2D synthetic seismic data using seismic unix.
- Migration of 2D synthetic seismic data using seismix unix.
- Forward modeling of 2D acoustic wave equation using finite differences method (second order in time and second order in space).
- Forward modeling of 2D acoustic wave equation using the pseudospectral method (second order in time, second, fourth and sixth order in space).

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Reverse Time Migration

- Forward propagation
- Backward propagation
- Condition image (cross-correlation)

$$I_{cc}(\mathbf{x}) = \int P_F(\mathbf{x}, t) P_B(\mathbf{x}, t) dt$$

Reverse Time Migration

Acoustic wave equation

$$\frac{1}{c^2}\frac{\partial^2 u(\mathbf{x},t)}{\partial t^2} - \nabla^2 u(\mathbf{x},t) = s(\mathbf{x},t) \tag{1}$$

$u(\mathbf{x},t)$: Wavefield at time t
$\mathbf{x} = (x,y,z)$: Position vector
$c = c(\mathbf{x})$: Acoustic propagation velocity
$s(\mathbf{x},t)$: Source term
$\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$: The Laplacian operator in Cartesian coordinates

Reverse Time Migration

Finite Difference

- 2D Forward propagation (Second order in time and space)

$$U_{i,j}^{n+1} = 2U_{i,j}^{n} - U_{i,j}^{n-1} + \nu^2[U_{i+1,j}^{n} + U_{i-1,j}^{n} + U_{i,j+1}^{n} + U_{i,j-1}^{n} - 4U_{i,j}^{n}] + S_{i,j}^{n} \tag{2}$$

with

$$\nu = \frac{c_{i,j}\Delta t}{h} \tag{3}$$

- 2D Backward propagation (Second order in time and space)

$$\widetilde{U}_{i,j}^{n+1} = 2\widetilde{U}_{i,j}^{n} - \widetilde{U}_{i,j}^{n-1} + \nu^2[\widetilde{U}_{i+1,j}^{n} + \widetilde{U}_{i-1,j}^{n} + \widetilde{U}_{i,j+1}^{n} + U_{i,j-1}^{n} - 4\widetilde{U}_{i,j}^{n}] + \widetilde{S}_{i,j}^{n} \tag{4}$$

Reverse Time Migration

Pseudospectral method

Rewriting the wave equation

$$\frac{\partial^2 u(\mathbf{x},t)}{\partial t^2} = -L^2 u(\mathbf{x},t) \tag{5}$$

with

$$-L^2 = c^2(\mathbf{x})\nabla^2$$

The formal solution of the equation 5 with initial conditions $\frac{\partial u(\mathbf{x},t)}{\partial t}(t = 0) = \dot{u}_0$ and $u(\mathbf{x}, t = 0) = u_0$ is given by

$$u(\mathbf{x},t) = \cos(Lt)u_0 + L^{-1}\sin(Lt)\dot{u}_0 \tag{6}$$

Reverse Time Migration

The wavefields $u(\mathbf{x}, t + \Delta t)$ and $u(\mathbf{x}, t - \Delta t)$ can be evaluated by equation 6. Adding these two wavefields result is

$$u(\mathbf{x}, t + \Delta t) + u(\mathbf{x}, t - \Delta t) = 2\cos(L\Delta t)u(\mathbf{x}, t) \tag{7}$$

If we take for $\cos(L\Delta t)$ its second-order $(1 - \frac{(L\Delta t)^2}{2})$ Taylor-series expansion, we obtain

$$u(\mathbf{x}, t + \Delta t) - 2u(\mathbf{x}, t) + u(\mathbf{x}, t - \Delta t) = -\Delta t^2 L^2 u(\mathbf{x}, t) \tag{8}$$

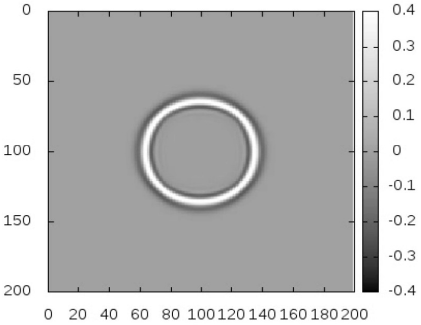
Reverse Time Migration

Using a pseudospectral method (Etgen, 1986 [24], Zhang et al., 2007 [68]) for the spatial derivatives, we can express equation (8) as:

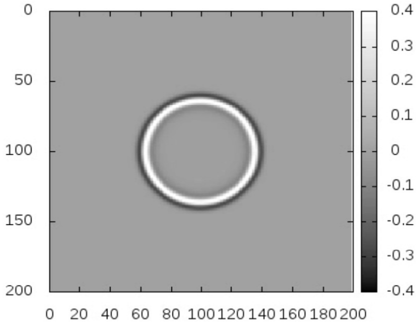
$$u(\mathbf{x}, t+\Delta t) - 2u(\mathbf{x}, t) + u(\mathbf{x}, t-\Delta t) = \Delta t^2 [c^2(\mathbf{x}) FT^{-1}(k_x^2 + k_y^2 + k_z^2) FT] u(\mathbf{x}, t)$$

Reverse Time Migration

Snapshot at 6 s



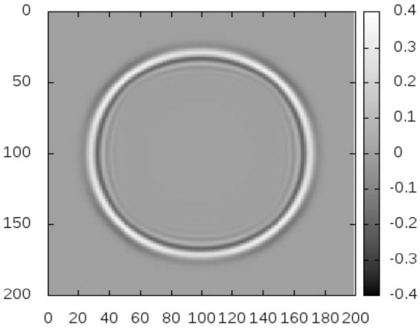
Finite difference



Pseudoespectral

Reverse Time Migration

Snapshot at 12 s



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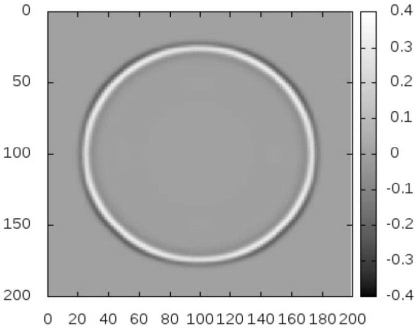


Pseudoespectral

Current works

Finite difference method

https://youtu.be/rhCRqaEHXqA

Pseudoespectral method

https://youtu.be/5MOmgKzpkx8

Current works

- C language implementation for Phase shift migration.

Phase shift Migration

2D acoustic wave equation

$$\frac{\partial^2 P}{\partial z^2} + \frac{\partial^2 P}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} = 0 \tag{9}$$

Applying the Fourier transform to the equation (9) we have

$$\frac{\partial^2 P}{\partial z^2} + \left(\frac{\omega^2}{c^2} - k_x^2\right) P = 0 \tag{10}$$

Let $A^2 = \frac{\omega^2}{c^2} - k_x^2$

Phase shift Migration

Then

$$\left(\frac{\partial^2}{\partial z^2} + A^2\right) P = 0 \tag{11}$$

$$\left(\frac{\partial}{\partial z} + iA\right)\left(\frac{\partial}{\partial z} - iA\right) P = 0 \tag{12}$$

If $c = c(z)$ but each subinterval $[z_i, z_{i+1}]$ is to be $c(z) = constant$, then we can solve the equation (12).

Then

$$\frac{\partial}{\partial z} P(k_x, z, \omega) = -i\sqrt{\frac{\omega^2}{c^2} - k_x^2} P(k_x, z, \omega) \tag{13}$$

Phase shift Migration

The equation to extrapolate the wavefield is

$$P(\omega, k_x, z + \Delta z) = P(\omega, k_x, z)e^{-ik_z \Delta z} \tag{14}$$

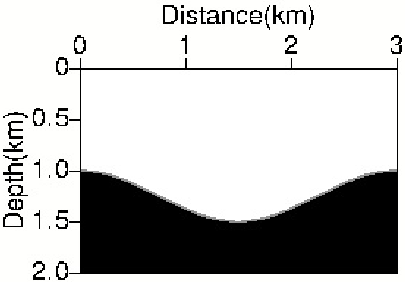
where

$$k_z = \sqrt{\frac{\omega^2}{c^2} - k_x^2}$$

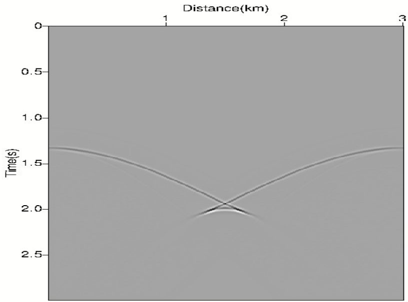
The migrated section on each $z + \Delta z$ level is given by (image condition at $t = 0$)

$$P(x, z + \Delta z) = \int d\omega \int P(\omega, k_x, z)e^{-ik_z \Delta z} e^{-ik_x x} dk_x \tag{15}$$

Phase shift Migration

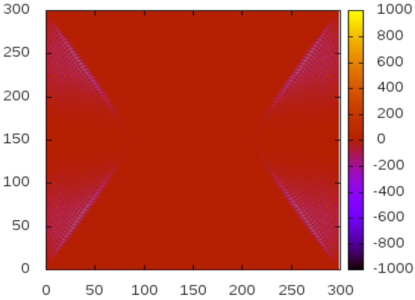


Sinclina model

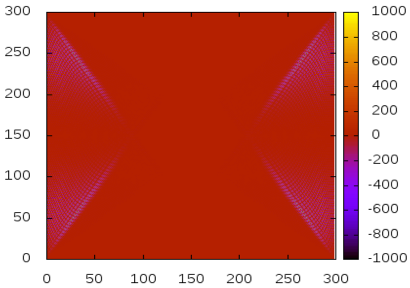


Seismic section

Phase shift Migration

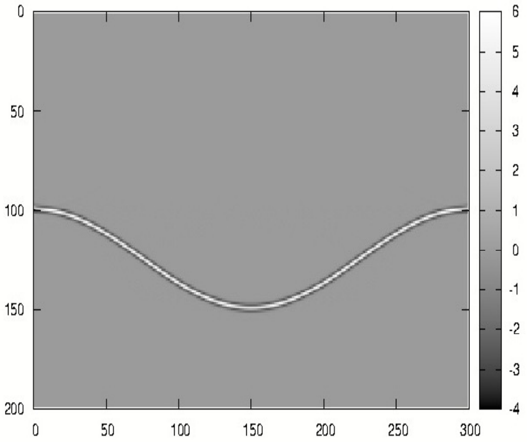


Real part of the spectrum



Imaginary part of the spectrum

Phase shift Migration



Phase shift Migration

Research question

Is it possible to implement Phase Shift Migration using discrete Haar transform instead of a Fourier transform?

Current works

- C language implementation for Phase shift migration
- The Haar system.
- Discrete approximation of a function using the Haar system.
- Discrete Haar Transform.

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Discrete Haar transform

Approximation and detail matrix

Given $L \in N$ even, define the $(L/2) \times L$ matrices H_L and G_L by

$$H_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & & \dots & & 0 \\ 0 & 0 & 1 & 1 & 0 & \dots & 0 \\ & & & \vdots & & & \\ 0 & & & \dots & 0 & 1 & 1 \end{pmatrix} \tag{16}$$

Discrete Haar transform

Approximation and detail matrix

$$G_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 & 0 & & \dots & & 0 \\ 0 & 0 & 1 & -1 & 0 & \dots & 0 \\ & & & \vdots & & & \\ 0 & & & \dots & 0 & 1 & -1 \end{pmatrix} \tag{17}$$

The matrix H_L is referred as the approximation matrix, the matrix G_L as the detail matrix.

Wavelet matrix

Define $L \times L$ matrix W_L by

$$W_L = \begin{pmatrix} H_L \\ G_L \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & & \dots & & 0 \\ 0 & 0 & 1 & 1 & 0 & \dots & 0 \\ & & & \vdots & & & \\ 0 & & & \dots & 0 & 1 & 1 \\ 1 & -1 & 0 & & \dots & & 0 \\ 0 & 0 & 1 & -1 & 0 & \dots & 0 \\ & & & \vdots & & & \\ 0 & & & \dots & 0 & 1 & -1 \end{pmatrix} \tag{18}$$

1D Discrete Haar transform

If we consider a initial sequence of data, a_0, to be a vector of length $L = 2^N$, $N \in \mathbf{N}$

$$a_0 = (a_0(0), a_0(1), \ldots, a_0(2^N - 1))$$

The discrete Haar transform (DHT) of a_0 is given by

$$\begin{pmatrix} c_j \\ d_j \end{pmatrix} = W_L a_0 = \begin{pmatrix} H_L \\ G_L \end{pmatrix} a_0 = \begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_{\frac{L}{2}-1} \\ d_0 \\ d_1 \\ \vdots \\ d_{\frac{L}{2}-1} \end{pmatrix}$$

1D Discrete Haar transform

c_j is called the averages block.

$$c_j = H_L a_0 = \begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_{\frac{L}{2}-1} \end{pmatrix}$$

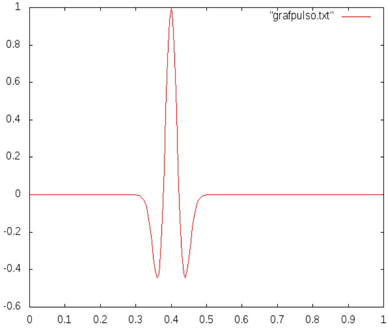
d_j is called the details block.

$$d_j = G_L a_0 = \begin{pmatrix} d_0 \\ d_1 \\ \vdots \\ d_{\frac{L}{2}-1} \end{pmatrix}$$

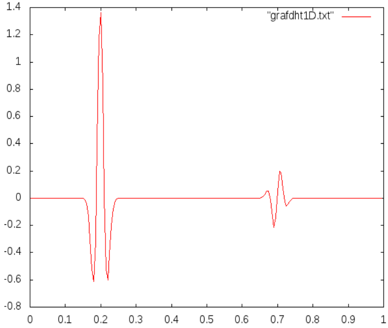
DHT Ricker Pulse

$$f(t) = (1 - 2\pi fc^2(t-0.4)^2)e^{-\pi^2 fc^2 (t-0.4)^2} \quad (19)$$

with $fc = 5\ hz$ y $0 \leq t < 1$



Original Ricker pulse



DHT Ricker pulse

2D Discrete Haar transform

2D Discrete Haar transform

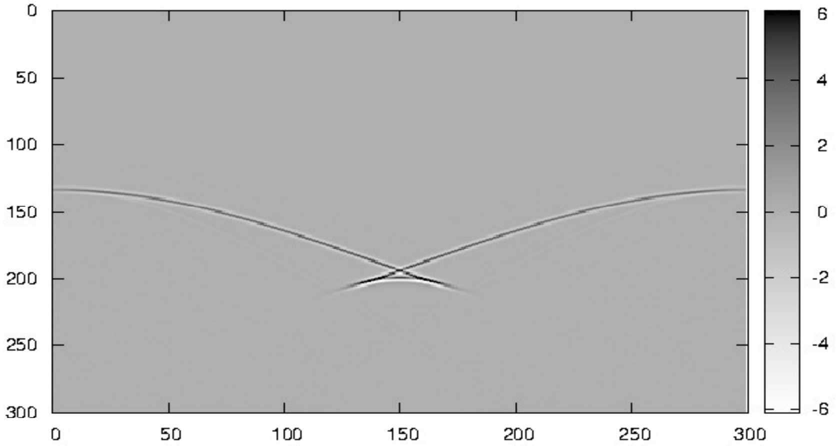
Suppose that A is an $M \times N$ matrix where M, N are even positive integers. The two-dimensional discrete Haar wavelet transformation of M x N matrix A is defined as

$$B = W_M \; A \; W_N^T \tag{20}$$

where W_M, W_N are defined by (18)

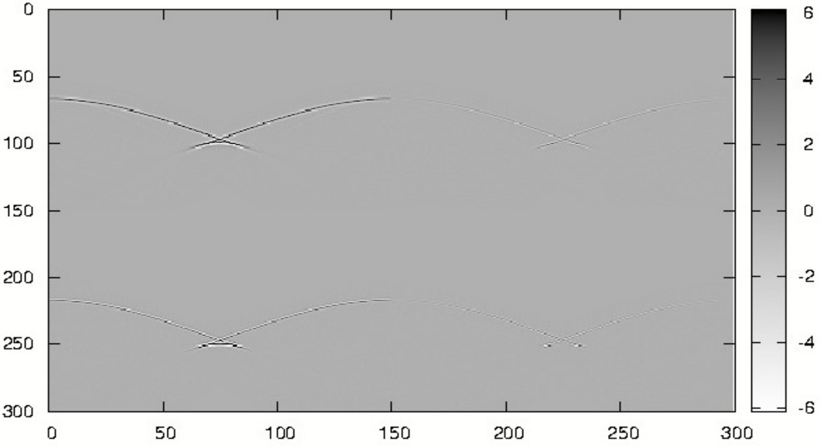
Sinclina model

Zero offset seismic section



DHT Sinclina model

DHT Zero offset seismic section



2D Discrete Haar transform

The transformed signal consists of four blocks, as follows:

$$\begin{aligned} B &= W_M \, A \, W_N^T \\ &= \begin{pmatrix} H_{\frac{M}{2}} \\ G_{\frac{M}{2}} \end{pmatrix} A \begin{pmatrix} H_{\frac{N}{2}} \\ G_{\frac{N}{2}} \end{pmatrix}^T \\ &= \begin{pmatrix} H_{\frac{M}{2}} \\ G_{\frac{M}{2}} \end{pmatrix} A \begin{pmatrix} H_{\frac{N}{2}}^T & G_{\frac{N}{2}}^T \end{pmatrix} \end{aligned}$$

2D Discrete Haar transform

$$
\begin{aligned}
&= \begin{pmatrix} H_{\frac{M}{2}} A \\ G_{\frac{M}{2}} A \end{pmatrix} \begin{pmatrix} H_{\frac{N}{2}}^T & G_{\frac{N}{2}}^T \end{pmatrix} \\
&= \begin{pmatrix} H_{\frac{M}{2}} A H_{\frac{N}{2}}^T & H_{\frac{M}{2}} A G_{\frac{N}{2}}^T \\ G_{\frac{M}{2}} A H_{\frac{N}{2}}^T & G_{\frac{M}{2}} A G_{\frac{N}{2}}^T \end{pmatrix} \\
B &= \begin{pmatrix} \mathcal{A} & \mathcal{V} \\ \mathcal{H} & \mathcal{D} \end{pmatrix}
\end{aligned}
$$

2D Discrete Haar transform

The upper left block is $\mathcal{A} = H_{\frac{M}{2}} A H_{\frac{N}{2}}^T$. $H_{\frac{M}{2}} A$ produces (weighted) column averages. We right-multiply this product by $H_{\frac{N}{2}}^T$ and this operation produces (weighted) averages along rows, so $\mathcal{A}$ is an approximation of the original input matrix A.

$\mathcal{V} = H_{\frac{M}{2}} A G_{\frac{N}{2}}^T$, $\mathcal{H} = G_{\frac{M}{2}} A H_{\frac{N}{2}}^T$ and $\mathcal{D} = G_{\frac{M}{2}} A G_{\frac{N}{2}}^T$ measure weighted differences in the column sums, row sums, and diagonal sums of A_{ij}, respectively.

And...

Is it possible to implement Phase Shift Migration using discrete Haar transform instead of a Fourier transform?

No. It is not possible to implement phase shift migration using the discrete Haar transform instead of a Fourier transform, because the DHT transforms the image into the same domain (space-time domain), resulting in a scaled and compressed image. In addition, input data in phase shift migration is used in the complex domain and the complex field is extrapolated for each level of depth.

Work perspectives

- Is it possible to calculate the Laplacian (2D, 3D) using a different method that does not use a finite differences scheme or pseudo-espectral?
- Is it possible to improve the dispersion in wave propagation using a different method to calculate the Laplacian (2D, 3D), for example, a wavelet transform or another transform?
- Is it feasible to find other 2D, 3D orthogonal transformation that can be used in another method of migration, such as reverse time migration, allowing improve the images obtained through the analysis of the signals recorded on the surface?
- Search for a methodology to find the subsurface velocity field (2D, 3D) through the analysis of the signals recorded on the surface.

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